Calculs élémentaires

Pour les BL, on admet pour $n \in \mathbf{N}$,

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

1 Collège - Lycée

Exercice 1

Simplifier au maximum.

$A = 7^{10} \times 7^3$ $\quad$ $C = \dfrac{3^{12}}{3^8}$ $\quad$ $E = (7^{10})^2 \times 7^{-4}$

$B = 5^4 \times 5^{-2}$ $\quad$ $D = \dfrac{3^7 \times 3^3}{3^8}$ $\quad$ $F = \dfrac{5^4}{5^{-2}}$

Exercice 2

Simplifier au maximum.

$A = 10^5 \times 10^2$ $\quad$ $D = 10^4 \times 5^{-2}$

$B = 10^{-8} \times 10^{-2} \times 10^4$ $\quad$ $E = (10^4)^3 \times 5^{-2}$

$C = \dfrac{10^5 \times 10^3}{10^8}$ $\quad$ $F = \dfrac{10^4}{2^2 \times 10^{-5}}$

Exercice 3

Factoriser et simplifier au maximum.

$A = \dfrac{a^{10} - a^8}{1 - a^2}$ $\quad$ $C = \dfrac{1}{a^{2d}} \dfrac{\left(ab^3\right)^d \left(a^{2d} - b^{2d}\right)}{a^d + b^d}$

$B = \dfrac{\left(a^2\right)^{10}}{\left(a^{10}\right)^2}$

Exercice 4

Simplifier au maximum.

$$\frac{\left(1 + \dfrac{x}{y}\right)^a \left(1 - \dfrac{y^2}{x^2}\right)^a}{(x+y)^{2a}}$$

2 Sommes simples

Exercice 5

1. $\displaystyle\sum_{k=1}^{n} 5$
2. $\displaystyle\sum_{k=1}^{n} 5k$
3. $\displaystyle\sum_{k=1}^{n} (5+k)$
4. $\displaystyle\sum_{k=1}^{n} (5-k)$
5. $\displaystyle\sum_{k=0}^{n} (2k+1)$
6. $\displaystyle\sum_{k=2}^{n} (3k+2)$
7. $\displaystyle\sum_{k=1}^{n-1} (k-1)$
8. $\displaystyle\sum_{k=1}^{n} \mathrm{e}^{ak}$
9. $\displaystyle\sum_{k=1}^{n} 2^{2k+1}$

Exercice 6

1. $\displaystyle\sum_{k=1}^{n} k^2$
2. $\displaystyle\sum_{k=0}^{n} k\,(2k+1)$
3. $\displaystyle\sum_{k=0}^{2n} (2k+1)$
4. $\displaystyle\left(\sum_{k=0}^{2n} 2k\right) + 1$

Exercice 7

Pour $n \geq 1$,

1. $\displaystyle\sum_{k=0}^{n} 3^k$
2. $\displaystyle\sum_{k=1}^{n} 3^k$
3. $\displaystyle\sum_{k=5}^{n} 3^k$
4. $\displaystyle\sum_{k=1}^{n} 3^{k+1}$
5. $\displaystyle\sum_{k=1}^{n} 3^{2k+1}$
6. $\displaystyle\sum_{k=1}^{n} 2^{-2k+1}$

Exercice 8

Pour $n \geq 1$,

1. $\displaystyle\sum_{k=1}^{n} \left(2^k - k^2\right)$
2. $\displaystyle\sum_{k=1}^{n} \left(2^{3k} - 3 \times 2^k\right)$
3. $\displaystyle\sum_{k=1}^{n} \left(2^{3k} - 2^3 \times 2^k\right)$
4. $\displaystyle\sum_{k=1}^{n} \left(2^{3k} - \left(2^k\right)^3\right)$
5. $\displaystyle\sum_{k=1}^{n} \left(2^{3k} - 3^{2k}\right)$
6. $\displaystyle\sum_{k=1}^{n} \left(2^{3k} - 3^2 k\right)$

3 Produits

Exercice 9

1. $\prod_{k=1}^{n} 2$
2. $\prod_{k=0}^{n} 2$
3. $\prod_{k=1}^{n} k$
4. $\prod_{k=0}^{n} k$
5. $\prod_{k=1}^{n} 2k$
6. $\prod_{k=1}^{n} k^2$
7. $\prod_{k=1}^{n} \sqrt{k}$
8. $\prod_{k=1}^{n} k\sqrt{k}$
9. $\prod_{k=1}^{n} \dfrac{1}{k(k+1)}$
10. $\prod_{k=1}^{n} k^{\sqrt{2}}$
11. $\prod_{k=1}^{n} 2^k$
12. $\prod_{k=1}^{n} 2^{2k+1}$
13. $\prod_{k=1}^{n} (-1)^k$
14. $\prod_{k=0}^{n-1} \mathrm{e}^{\frac{2ik\pi}{n}}$

4 Coefficients binomiaux

Exercice 10

Calculer les sommes suivantes :

1. $\sum_{k=0}^{n} \binom{n}{k} 3^k$
2. $\sum_{k=0}^{n} \binom{n}{k} 2^{n-k}$
3. $\sum_{k=1}^{n} \binom{n}{k} 3^{k+1}$
4. $\sum_{k=0}^{n-1} \binom{n-1}{k} 3^{k-1}$
5. $\sum_{k=0}^{n-2} \binom{n-1}{k} 3^{2k-1}$
6. $\sum_{k=0}^{n} \binom{n}{k} \dfrac{3^{k-1}}{2^{2k}}$
7. $\sum_{k=1}^{n} \binom{n}{k} \dfrac{2^k}{3^{k+1}}$
8. $\sum_{k=0}^{n-1} \binom{n}{k+1} \dfrac{2^n(-1)^{k(k+1)}}{3^{2k}}$

5 Sommes multiples

Exercice 11

Calculer les sommes suivantes

1. $\sum_{0\leq i<j\leq n} 1$
2. $\sum_{1\leq i<j<k\leq n} 1$
3. $\sum_{0\leq j<i\leq n} 2^j$
4. $\sum_{1\leq i,j\leq n} (i+j)$
5. $\sum_{1\leq i,j\leq n} (i-j)$
6. $\sum_{i=0}^{n}\sum_{k=i}^{n} \dfrac{i}{k+1}$

6 Divers

Exercice 12

1. $\sum_{k=1}^{n} \left(\dfrac{1}{k} - \dfrac{1}{k+1}\right)$
2. $\sum_{k=1}^{n} \ln\left(\dfrac{k+1}{k}\right)$
3. $\sum_{k=1}^{n} \dfrac{1}{k(k+1)}$
4. $\sum_{k=1}^{n} \dfrac{1}{k(k+2)}$
5. $\sum_{k=2}^{n} \dfrac{1}{(k-1)(k+1)}$
6. $\sum_{k=2}^{n} \dfrac{1}{k^2+2k-3}$

Exercice 13

1. $\prod_{k=1}^{n} (k+1)$
2. $\prod_{k=1}^{n} (k+1)^3$
3. $\prod_{k=1}^{n} (2k+1)$
4. $\prod_{k=2}^{n} \sqrt{k^2-4}$
5. $\prod_{k=3}^{n} \sqrt{k^2-4}$
6. $\prod_{k=1}^{n} \dfrac{3k}{k^2+6k+9}$
7. $\prod_{k=2}^{n} \dfrac{k^2+k}{k^2+k-2}$