Équations - inéquations

Résoudre les équations et inéquations suivantes sur $\mathbf{R}$.

1 Cas simples

Exercice 1

1. $x^3 + 4x = x^2$
2. $(x-3)^3 = 8$
3. $x(x-1) = (x-1)^2$
4. $x^4 - 4 = (x^2-2)^2$

Exercice 2

1. $x^2 \leq 1$
2. $x(x+2) \leq 3$
3. $(x+3)^4 \leq 4$
4. $x^3 + x^2 \leq x + 1$
5. $x(x-1) - 2 \geq 8(x-2)$
6. $(x-1)^2 < x - 1$

Exercice 3

1. $\dfrac{x+1}{x+2} \leq 0$
2. $\dfrac{x+1}{x+2} \leq (x+2)$
3. $\dfrac{x^2+6x+4}{x} \leq 2$

Exercice 4

1. $2x^4 - 8x^3 + 12x^2 - 8x = -2$
2. $x^3 - 2x - 1 = 0$
3. $x^3 + 2x^2 - 15x = 0$

2 Valeurs absolues

Exercice 5

1. $|x+1| = |x-1|$
2. $|x^2+x| = |x+1|$
3. $|x^2+x+1| = |x+1|$
4. $|x^2+x+1| = x+1$

Exercice 6

1. $|x+1| \leq 2$
2. $|x^2+x| > 0$
3. $|x^2+2x-2| \leq 1$
4. $|x^2+2x-1| \geq 1$
5. $|3x^2+7x+4| \leq 3x^2+5x+10$
6. $3x^2+7x+4 \leq |3x^2+5x+10|$

3 Divers

Exercice 7

1. $\sqrt{2x+4} = x$
2. $\sqrt{2x+4} \geq x$
3. $\sqrt{x^2+2x-1} = \sqrt{x-2}$
4. $\ln(x^2+x-1) \geq 0$
5. $\ln\left(x^2-6x+9\right) = 2\ln(x-3)$
6. $x^{\sqrt{x}} = (\sqrt{x})^x$

Exercice 8

1. $\mathrm{e}^x + \mathrm{e}^{-x} = 2$
2. $\mathrm{e}^x + \mathrm{e}^{-x} > 0$
3. $\mathrm{e}^{x^2} - \mathrm{e}^{-x} > 0$
4. $\mathrm{e}^x + \mathrm{e}^{2x} > 2$
5. $\mathrm{e}^x > x + 1$

Exercice 9

1. $\lfloor x \rfloor = x$
2. $\left\lfloor \sqrt{x^2+1} \right\rfloor = 2$
3. $\lfloor \sqrt{x} \rfloor = \sqrt{\lfloor x \rfloor}$

Exercice 10

1. $\begin{cases} e^x\,\mathrm{e}^{2y} = a \\ 2xy = 1 \end{cases}$
2. $\begin{cases} 2\ln_x y + 2\ln_y x = -5 \\ xy = e \end{cases}$