Formulaire de trigonométrie

Cosinus

$$\begin{array}{ccc} \mathbf{R} & \xrightarrow{\cos} & [-1,1] \\ [0,\pi] & \xleftarrow[\text{Arccos}]{} & [-1,1] \end{array}$$

$$-1 \leqslant \cos\,\theta \leqslant 1$$

cos est 2π-périodique :

$$\cos\,(\theta+2\pi) = \cos\,\theta$$
$$\forall k \in \mathbf{Z},\ \cos\,(\theta+2k\pi) = \cos\,\theta$$

cos est *pair* : $\cos\,(-\theta) = \cos\,\theta$

$$\cos(a+b) = \cos a\cos b - \sin a\sin b$$
$$\cos(a-b) = \cos a\cos b + \sin a\sin b$$

$$\cos(2a) = 2\cos^2 a - 1 = 1 - 2\sin^2 a$$

Sinus

$$\begin{array}{ccc} \mathbf{R} & \xrightarrow{\sin} & [-1,1] \\ \left[-\frac{\pi}{2},\frac{\pi}{2}\right] & \xleftarrow[\text{Arcsin}]{} & [-1,1] \end{array}$$

$$-1 \leqslant \sin\,\theta \leqslant 1$$

sin est 2π-périodique :

$$\sin\,(\theta+2\pi) = \sin\,\theta$$
$$\forall k \in \mathbf{Z},\ \sin\,(\theta+2k\pi) = \sin\,\theta$$

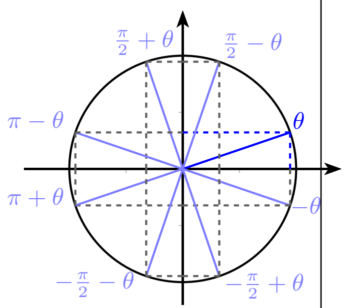
sin est *impair* : $\sin\,(-\theta) = -\sin\,\theta$

$$\sin(a+b) = \sin a\cos b + \cos a\sin b$$
$$\sin(a-b) = \sin a\cos b - \cos a\sin b$$

$$\sin(2a) = 2\sin a\cos a$$

$$\cos\,(\pi-\theta) = -\cos\,\theta$$
$$\cos\,(\pi+\theta) = -\cos\,\theta$$
$$\cos\left(\frac{\pi}{2}-\theta\right) = \sin\,\theta$$
$$\cos\left(\frac{\pi}{2}+\theta\right) = -\sin\,\theta$$

$$\sin\,(\pi-\theta) = \sin\,\theta$$
$$\sin\,(\pi+\theta) = -\sin\,\theta$$
$$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\,\theta$$
$$\sin\left(\frac{\pi}{2}+\theta\right) = \cos\,\theta$$



Pythagore :

$$\forall \theta \in \mathbf{R}, \qquad \cos^2\theta + \sin^2\theta = 1$$

Formules de linéarisation :

$$\cos a\cos b = \frac{1}{2}\big(\cos(a+b)+\cos(a-b)\big),$$

$$\sin a\sin b = \frac{1}{2}\big(\cos(a-b)-\cos(a+b)\big), \quad \sin a\cos b = \frac{1}{2}\big(\sin(a+b)+\sin(a-b)\big).$$

Formules de factorisation :

$$\cos p + \cos q = 2\cos\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right), \quad \cos p - \cos q = -2\sin\left(\frac{p+q}{2}\right)\sin\left(\frac{p-q}{2}\right),$$

$$\sin p + \sin q = 2\sin\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right), \quad \sin p - \sin q = 2\cos\left(\frac{p+q}{2}\right)\sin\left(\frac{p-q}{2}\right).$$

Tangente

$$\begin{array}{ccc} \mathbf{R}\setminus\left\{\frac{\pi}{2}+k\pi,\ k\in\mathbf{Z}\right\} & \xrightarrow{\tan} & \mathbf{R} \\ \left]-\frac{\pi}{2},\frac{\pi}{2}\right[& \xleftarrow[\text{Arctan}]{} & \mathbf{R} \end{array}$$

tan est π-périodique :

$$\tan\,(\theta+\pi) = \tan\,\theta, \qquad \forall k \in \mathbf{Z},\ \tan\,(\theta+k\pi) = \tan\,\theta.$$

tan est *impaire* : $\tan\,(-\theta) = -\tan\,\theta$.

$$\tan\,(\theta-\pi) = -\tan\,\theta, \qquad \tan\,(\theta+\pi) = \tan\,\theta.$$

$$\tan\,(a+b) = \frac{\tan\,(a)+\tan\,(b)}{1-\tan\,(a)\tan\,(b)}.$$

Formules sin-cos-tan :

$$\tan\,(\theta) = \frac{\sin\,(\theta)}{\cos\,(\theta)}, \qquad 1+\tan^2(\theta) = \frac{1}{\cos^2(\theta)}.$$

Si $t = \tan\left(\frac{\theta}{2}\right)$, alors

$$\cos\,(\theta) = \frac{1-t^2}{1+t^2}, \quad \sin\,(\theta) = \frac{2t}{1+t^2}, \quad \tan\,(\theta) = \frac{2t}{1-t^2}.$$

Fonctions réciproques :

$$\forall x \in [-1,\,1]\,,\ \text{Arccos}\,(x)+\text{Arcsin}\,(x) = \frac{\pi}{2} \qquad \forall x \in \mathbf{R}^*,\ \text{Arctan}\,(x)+\text{Arctan}\left(\frac{1}{x}\right) = \frac{\pi}{2}\text{sgn}(x).$$

Angles remarquables :

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\cos\,\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\sin\,\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\tan\,\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\times$