Formulaire de trigonométrie

Cosinus

$$\mathbf{R} \xrightarrow{\cos} [-1,1] \qquad [0,\pi] \xleftarrow[\arccos]{} [-1,1]$$

$$-1 \leqslant \cos\theta \leqslant 1$$

cos est 2π-périodique :

$$\cos(\theta+2\pi) = \cos\theta$$
$$\forall k \in \mathbf{Z},\ \cos(\theta+2k\pi) = \cos\theta$$

cos est *pair* : $\cos(-\theta) = \cos\theta$

$$\cos(a+b) = \cos a\cos b - \sin a\sin b$$
$$\cos(a-b) = \cos a\cos b + \sin a\sin b$$

$$\cos(2a) = 2\cos^2 a - 1 = 1 - 2\sin^2 a$$

Sinus

$$\mathbf{R} \xrightarrow{\sin} [-1,1] \qquad \left[-\tfrac{\pi}{2},\tfrac{\pi}{2}\right] \xleftarrow[\arcsin]{} [-1,1]$$

$$-1 \leqslant \sin\theta \leqslant 1$$

sin est 2π-périodique :

$$\sin(\theta+2\pi) = \sin\theta$$
$$\forall k \in \mathbf{Z},\ \sin(\theta+2k\pi) = \sin\theta$$

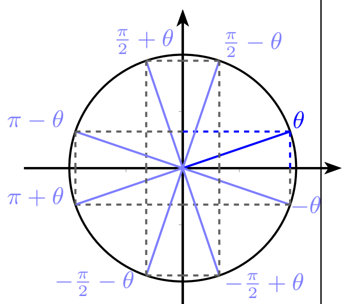
sin est *impair* : $\sin(-\theta) = -\sin\theta$

$$\sin(a+b) = \sin a\cos b + \cos a\sin b$$
$$\sin(a-b) = \sin a\cos b - \cos a\sin b$$

$$\sin(2a) = 2\sin a\cos a$$

$$\cos(\pi-\theta) = -\cos\theta \qquad \cos(\pi+\theta) = -\cos\theta$$
$$\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta \qquad \cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$$

$$\sin(\pi-\theta) = \sin\theta \qquad \sin(\pi+\theta) = -\sin\theta$$
$$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta \qquad \sin\left(\frac{\pi}{2}+\theta\right) = \cos\theta$$



Pythagore :

$$\forall\theta\in\mathbf{R},\qquad \cos^2\theta + \sin^2\theta = 1$$

Formules de linéarisation :

$$\cos a\cos b = \frac{1}{2}\big(\cos(a+b)+\cos(a-b)\big),$$

$$\sin a\sin b = \frac{1}{2}\big(\cos(a-b)-\cos(a+b)\big),\quad \sin a\cos b = \frac{1}{2}\big(\sin(a+b)+\sin(a-b)\big).$$

Formules de factorisation :

$$\cos p+\cos q = 2\cos\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right),\quad \cos p-\cos q = -2\sin\left(\frac{p+q}{2}\right)\sin\left(\frac{p-q}{2}\right),$$

$$\sin p+\sin q = 2\sin\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right),\quad \sin p-\sin q = 2\cos\left(\frac{p+q}{2}\right)\sin\left(\frac{p-q}{2}\right).$$

Tangente

$$\mathbf{R}\setminus\left\{\tfrac{\pi}{2}+k\pi,\ k\in\mathbf{Z}\right\} \xrightarrow{\tan} \mathbf{R} \qquad \left]-\tfrac{\pi}{2},\tfrac{\pi}{2}\right[\xleftarrow[\arctan]{} \mathbf{R}$$

tan est π-périodique :

$$\tan(\theta+\pi) = \tan\theta,\qquad \forall k\in\mathbf{Z},\ \tan(\theta+k\pi) = \tan\theta.$$

tan est *impaire* : $\tan(-\theta) = -\tan\theta$.

$$\tan(\theta-\pi) = -\tan\theta,\qquad \tan(\theta+\pi) = \tan\theta.$$

$$\tan(a+b) = \frac{\tan(a)+\tan(b)}{1-\tan(a)\tan(b)}.$$

Formules sin-cos-tan :

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)},\qquad 1+\tan^2(\theta) = \frac{1}{\cos^2(\theta)}.$$

Si $t = \tan\left(\frac{\theta}{2}\right)$, alors

$$\cos(\theta) = \frac{1-t^2}{1+t^2},\quad \sin(\theta) = \frac{2t}{1+t^2},\quad \tan(\theta) = \frac{2t}{1-t^2}.$$

Fonctions réciproques :

$$\forall x\in[-1,\,1],\ \arccos(x)+\arcsin(x) = \frac{\pi}{2}\qquad \forall x\in\mathbf{R}^*,\ \arctan(x)+\arctan\left(\frac{1}{x}\right) = \frac{\pi}{2}\mathrm{sgn}(x).$$

Angles remarquables :

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\times$