

CALCULS ÉLÉMENTAIRES

Pour les BL, on admet pour $n \in \mathbf{N}$,

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

1 COLLÈGE - LYCÉE

Exercice 1

Simplifier au maximum.

$$\begin{aligned} A &= 7^{10} \times 7^3 & C &= \frac{3^{12}}{3^8} & E &= (7^{10})^2 \times 7^{-4} \\ B &= 5^4 \times 5^{-2} & D &= \frac{3^7 \times 3^3}{3^8} & F &= \frac{5^4}{5^{-2}} \end{aligned}$$

Exercice 2

Simplifier au maximum.

$$\begin{aligned} A &= 10^5 \times 10^2 & D &= 10^4 \times 5^{-2} \\ B &= 10^{-8} \times 10^{-2} \times 10^4 & E &= (10^4)^3 \times 5^{-2} \\ C &= \frac{10^5 \times 10^3}{10^8} & F &= \frac{10^4}{2^2 \times 10^{-5}} \end{aligned}$$

Exercice 3

Factoriser et simplifier au maximum.

$$\begin{aligned} A &= \frac{a^{10} - a^8}{1 - a^2} & C &= \frac{1}{a^{2d}} \frac{(ab^3)^d (a^{2d} - b^{2d})}{a^d + b^d} \\ B &= \frac{(a^2)^{10}}{(a^{10})^2} \end{aligned}$$

Exercice 4

Simplifier au maximum.

$$\frac{\left(1 + \frac{x}{y}\right)^a \left(1 - \frac{y^2}{x^2}\right)^a}{(x+y)^{2a}}$$

2 SOMMES SIMPLES

Exercice 5

$$\begin{aligned} 1. \sum_{k=1}^n 5 & & 5. \sum_{k=0}^n (2k+1) & & 8. \sum_{k=1}^n e^{ak} \\ 2. \sum_{k=1}^n 5k & & 6. \sum_{k=2}^n (3k+2) & & 9. \sum_{k=1}^n 2^{2k+1} \\ 3. \sum_{k=1}^n (5+k) & & 7. \sum_{k=1}^{n-1} (k-1) & & 10. \sum_{k=1}^n \ln\left(\frac{k+1}{k}\right) \\ 4. \sum_{k=1}^n (5-k) & & & & \end{aligned}$$

Exercice 6

$$\begin{aligned} 1. \sum_{k=1}^n k^2 & & 3. \sum_{k=0}^{2n} (2k+1) \\ 2. \sum_{k=0}^n k(2k+1) & & 4. \left(\sum_{k=0}^{2n} 2k\right) + 1 \end{aligned}$$

Exercice 7

Pour $n \geq 1$,

$$\begin{aligned} 1. \sum_{k=0}^n 3^k & & 3. \sum_{k=5}^n 3^k & & 5. \sum_{k=1}^n 3^{2k+1} \\ 2. \sum_{k=1}^n 3^k & & 4. \sum_{k=1}^n 3^{k+1} & & 6. \sum_{k=1}^n 2^{-2k+1} \end{aligned}$$

Exercice 8

Pour $n \geq 1$,

$$\begin{aligned} 1. \sum_{k=1}^n (2^k - k^2) & & 4. \sum_{k=1}^n (2^{3k} - (2^k)^3) \\ 2. \sum_{k=1}^n (2^{3k} - 3 \times 2^k) & & 5. \sum_{k=1}^n (2^{3k} - 3^{2k}) \\ 3. \sum_{k=1}^n (2^{3k} - 2^3 \times 2^k) & & 6. \sum_{k=1}^n (2^{3k} - 3^{2k}) \end{aligned}$$

3 PRODUITS

Exercice 9

1. $\prod_{k=1}^n 2$
2. $\prod_{k=0}^n 2$
3. $\prod_{k=1}^n k$
4. $\prod_{k=0}^n k$
5. $\prod_{k=1}^n 2k$
6. $\prod_{k=1}^n k^2$
7. $\prod_{k=1}^n \sqrt{k}$
8. $\prod_{k=1}^n k\sqrt{k}$
9. $\prod_{k=1}^n \frac{1}{k(k+1)}$
10. $\prod_{k=1}^n k^{\sqrt{2}}$
11. $\prod_{k=1}^n 2^k$
12. $\prod_{k=1}^n 2^{2k+1}$
13. $\prod_{k=1}^n (-1)^k$
14. $\prod_{k=0}^{n-1} e^{\frac{2ik\pi}{n}}$

4 COEFFICIENTS BINOMIAUX

Exercice 10

Calculer les sommes suivantes :

1. $\sum_{k=0}^n \binom{n}{k} 3^k$
2. $\sum_{k=0}^n \binom{n}{k} 2^{n-k}$
3. $\sum_{k=1}^n \binom{n}{k} 3^{k+1}$
4. $\sum_{k=0}^{n-1} \binom{n-1}{k} 3^{k-1}$
5. $\sum_{k=0}^{n-2} \binom{n-1}{k} 3^{2k-1}$
6. $\sum_{k=0}^n \binom{n}{k} \frac{3^{k-1}}{2^{2k}}$
7. $\sum_{k=1}^n \binom{n}{k} \frac{2^k}{3^{k+1}}$
8. $\sum_{k=0}^{n-1} \binom{n}{k+1} \frac{2^n (-1)^{k(k+1)}}{3^{2k}}$

5 SOMMES MULTIPLES

Exercice 11

Calculer les sommes suivantes

1. $\sum_{0 \leq i < j \leq n} 1$
2. $\sum_{1 \leq i < j < k \leq n} 1$
3. $\sum_{0 \leq j < i \leq n} 2^j$
4. $\sum_{1 \leq i, j \leq n} (i+j)$
5. $\sum_{1 \leq i, j \leq n} (i-j)$
6. $\sum_{i=0}^n \sum_{k=i}^n \frac{i}{k+1}$

6 DIVERS

Exercice 12

1. $\sum_{k=1}^n \left(\frac{1}{k} - \frac{1}{k+1} \right)$
2. $\sum_{k=1}^n \ln \left(\frac{k+1}{k} \right)$
3. $\sum_{k=1}^n \frac{1}{k(k+1)}$
4. $\sum_{k=1}^n \frac{1}{k(k+2)}$
5. $\sum_{k=2}^n \frac{1}{(k-1)(k+1)}$
6. $\sum_{k=2}^n \frac{1}{k^2 + 2k - 3}$

Exercice 13

1. $\prod_{k=1}^n (k+1)$
2. $\prod_{k=1}^n (k+1)^3$
3. $\prod_{k=1}^n (2k+1)$
4. $\prod_{k=2}^n \sqrt{k^2 - 4}$
5. $\prod_{k=3}^n \sqrt{k^2 - 4}$
6. $\prod_{k=1}^n \frac{3k}{k^2 + 6k + 9}$
7. $\prod_{k=2}^n \frac{k^2 + k}{k^2 + k - 2}$